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GLUONIUM AND SCALAR MESONS

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The inclusion of gluonia into the quark model of superconductivity type is shown. The possibility of the identification of arising three isoscalar states with the scalar mesons (σ, S*, ϵ) is discussed.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Глюоний и скалярные мезоны

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Производится включение глюониев в кварковую модель сверхпроводящего типа и обсуждается возможность отождествления возникнувших трех изоскалярных состояний со скалярными мезонами (σ, S*, ϵ).

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One of the problems, appearing in the description of scalar mesons in the framework of the quark models, is the problem how to interpret the S*(975) and ϵ(1300) states (according to the new notation f_0(975) and f_0(1300), respectively). These mesons are often interpreted as the mixing of singlet and octet quarkonium states with the primary content of s-quarks in the S*(975) state and of u, d quarks in the ϵ(1300) state. There are many questions arising, and it is difficult to answer most of them in the framework of pure quark models.

Firstly, why the ϵ(1300) state, consisting mainly of light quarks, is heavier than the S*(975) state, consisting mainly of the s quarks? The second question is connected with the problem of existence of the third isoscalar meson which should be lighter than 1 GeV. The existence of this meson has been predicted in the framework of phenomenological chiral sigma models many years ago. This isoscalar meson is well known as the σ-meson. The model value of its mass is from the region 500-700 MeV. The meson in question plays very important role as an intermediate state absolutely necessary for the correct description of such processes as, e.g., π-π scattering, $\eta' \to \eta 2\pi$ decay, pion

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polarizability, etc., in the framework of linear σ-model. Let us remind that linear σ-model appears quite naturally from the group as well as quark approaches as the universal model allowing one to describe the scalar, pseudoscalar, vector and axial-vector mesons [1,2]. However, the experimental status of ϵ (700) meson remains for the present as not completely unique. It is absent until now in the data [3] though possible to be observed in the indirect way, for example, in the $Q_1(1200) \to K\epsilon$ and $Q_2(1400) \to K\epsilon$ decays shown in the above-mentioned data. This is due to the fact that the discussed resonance has a very large width almost equal to its mass, and that is why it is hardly observable in the pure form [4]. Nevertheless, as has been mentioned before, it plays a very important role as an intermediate state, e.g., in $\pi-\pi$ scattering,where its width is small.

In the present paper it will be shown that if we generalize the earlier proposed quark model of the superconductivity type [1] in such a way that this model will include the gluonium, then the three scalar states (ϵ (550), $S^*(1070)$ and ϵ' (1200)) will appear in the natural way, and they can be identified with mesons we discuss in this paper. There is first resonance consisting mainly of light quarks and possessing the large width equal to its mass. The second state contains basically the s-quarks and possesses the small width of decay into 2π. And finally, the third, heavy state ϵ' (1200) is almost completely defined by the gluonium and it decays mainly into 2π and with smaller probability also into 2K. We have actually obtained a qualitatively reasonable picture expressing the existence of three such states in the complete agreement with experimental data.

The inclusion of the gluonia into the quark model of the superconductivity type [1] we shall carry out in the following scheme proposed in papers [5,6]. The σ-model Lagrangian, written in the chiral invariant form (till the spontaneous chiral symmetry breaking occurred) is expressed as follows:

$$\mathfrak{L}(\sigma', \phi) = \frac{1}{2}[(\partial_\mu \sigma'_a)^2 + (\partial_\mu \phi_a)^2] - \frac{1}{2}\mathrm{Tr}[\mu_a^2 (\bar{\sigma}'^2 + \bar{\phi}^2)] - \tag{1}$$
$$- \frac{1}{4}\mathrm{Tr}\{g_a^2([\bar{\sigma}'^2 + \bar{\phi}^2]^2 - [\bar{\sigma}', \bar{\phi}]_-^2)\} - \frac{1}{2G_1}\mathrm{Tr}\{g_a M \circ \bar{\sigma}'\},$$

where $\sigma'_a(\phi_a)$ denote the scalar (pseudoscalar) meson fields, $\bar{\sigma}(\bar{\phi}) = \sigma_a \lambda^a$ $(\phi_a \lambda^a)$, λ^a are the Gell-Mann matrices $(0 \le a \le 8)$, $\bar{\sigma}' = \bar{\sigma} - (M/g_a) = \bar{\sigma} - Z^{1/2} f_a$ $(\langle 0|\bar{\sigma}'|0\rangle = -(M/g_a))$, M is the mass matrix of constituent quarks ($M = \mathrm{diag}(m_u, m_d, m_s)$), $g_a = m_a/f_a Z^{1/2}$, f_a are meson decay constants ($f_{1,2,3,u} = f_\pi = 93$ MeV, $f_{4,5,6,7} = f_K =$

$1.16\, f_\pi$, $f_s = 1.28\, f_\pi$), $Z = 1.4$ is the constant occurring from the inclusion of π-A_1 transitions [1], $\mu_u^2 = (m_\pi^2/Z) - 2m_u^2$, $\mu_s^2 = (m_{\eta_s}^2/Z) - 2m_s^2$ ($m_{\eta_s} = 700$ MeV), $\mu_K^2 = (m_K^2/Z) - (m_u + m_s)^2/2$, M^0 is the matrix corresponding to the current quark masses, and finally, $G_1 = 4.9\ \mathrm{GeV}^{-2}$ is the four-quark interaction constant ($m_u = 280$ MeV, $m_s = 455$ MeV).

The gluonium field $\bar{G}$ we introduce in the following way [5,6]

$$\mathcal{L}(\sigma', \phi, \bar{G}) = \frac{1}{2}(\partial_\mu \bar{G})^2 + \frac{1}{2}[(\partial_\mu \sigma'_a)^2 + (\partial_\mu \phi_a)^2] - V(\bar{G}, \sigma', \phi), \qquad (2)$$

where

$$V(\bar{G}, \sigma', \phi) = H_0 \left(\frac{\bar{G}}{f_g}\right)^4 \ln\frac{\bar{G}}{C} + \bar{G}^4\, \mho\left(\sqrt{\frac{\sigma'^2_a + \phi_a^2}{\bar{G}}}\right)$$

$H_0 = \frac{b}{8} G_0$, $b = \frac{11}{3} N_c - \frac{2}{3} N_F$ ($N_c = 3$ is the number of colours, $N_F = 3$ is the flavour number), $G_0 = \langle 0 | (\alpha_s/\pi)\, G^a_{\mu\nu}\, G_a^{\mu\nu} | 0 \rangle$ is the gluon condensate for which we use following value $G_0 = 0.012\ \mathrm{GeV}^4$ according to [7], C is an unidentified parameter, $f_g = \langle 0 | \bar{G} | 0 \rangle$ and $\mho$ is the chiral symmetric part of Lagrangian (1) without kinetic terms. Similarly to [5,6] we shall also use the exponential representation for the gluonium field

$$\bar{G}(x) = f_g \exp(G(x)/f_g)\,. \qquad (3)$$

Then, we obtain from (1) and (2) the following expression for the Lagrangian describing the interaction of the gluonium with quarkonium fields (of scalar and pseudoscalar mesons)

$$\Delta\mathcal{L} = -H_0\, e^{4G/f_g} \left(\ln\frac{f_g}{C} + \frac{G}{f_g}\right) +$$

$$+ (e^{2G/f_g} - 1)\{(m_u^2 - \frac{m_\pi^2}{2Z})[(\vec{\pi}^2 + \eta_u^2) Z + (\sigma_u - Z^{1/2} f_\pi)^2] + \qquad (4)$$

$$+ (m_s^2 - \frac{m^2_{\eta_s}}{2Z})[\eta_s^2 Z + (\sigma_s - Z^{1/2} f_s)^2] + [(\frac{m_u + m_s}{2})^2 Z - \frac{m_K^2}{2}]\, \vec{K}^2\}\,.$$

In the Lagrangian (4) there are three new uncertain parameters C, f_g and m_G (gluonium mass); for the fixation of them it is necessary to choose three conditions. The first of them, which is enough for the determination of the parameter C, is the condition for the lack of the linear (in G) terms in (4). It leads to the equation

$$4H_0\left(\ln\frac{f_g}{C}+\frac{1}{4}\right) = (2m_u^2 Z - m_\pi^2) f_\pi^2 + (2m_s^2 Z - m_{\eta_s}^2) f_s^2 . \qquad (5)$$

The equation for the gluonium mass m_G follows from (4) and it allows one to fix the combination $(f_g m_G)^2$

$$f_g^2 m_G^2 = 4H_0 + 2[(2m_u^2 Z - m_\pi^2) f_\pi^2 + (2m_s^2 Z - m_{\eta_s}^2) f_s^2] = $$
$$= (0.054 + 0.006 = 0.06)\ \mathrm{GeV}^4 . \qquad (6)$$

To determine the third parameter f_g we use the experimental value of the $S^* \to 2\pi$ decay width: $\Gamma_{S^*\to 2\pi} = 26$ MeV. Let us remind that the Lagrangian (1) brings to the ideal mixing of singlet — octet states, as a result of which we get the following states: σ_u, consisting only of u and d quarks ($m^2_{\sigma_u} = m_\pi^2/Z + 4m_u^2$, $m_{\sigma_u} = 570$ MeV), and σ_s, consisting only of the s quarks ($m^2_{\sigma_s} = m^2_{\eta_s}/Z + 4m_s^2$, $m_{\sigma_s} = 1090$ MeV). The first state decays into two pions with high probability /1/, and the decay into 2π of the second state is forbidden.

However, with the appearance of the Lagrangian (4) the new situation arises. In (4) there firstly appear the nondiagonal terms, containing $G\sigma_u$ and $G\sigma_s$,

$$\mathcal{L}(G\sigma_u, G\sigma_s) = -\frac{2G}{Z^{1/2} f_g}[(2m_u^2 Z - m_\pi^2) f_\pi \sigma_u + (2m_s^2 Z - m_{\eta_s}^2) f_s \sigma_s]. \qquad (7)$$

Secondly, the direct decay of the gluonium into 2π and 2K is allowed

$$\mathcal{L}(G\vec{\pi}^2, G\vec{K}^2) = \frac{G}{f_g}\{(2m_u^2 Z - m_\pi^2)\vec{\pi}^2 + (\frac{(m_u + m_s)^2}{2} Z - m_K^2)\vec{K}^2]. \qquad (8)$$

As a result, the possibility of the $\sigma_s \to 2\pi$ decay via the intermediate G-state opens

$$\mathcal{L}^{eff}(\sigma_s \vec{\pi}^2) = i\,\mathcal{L}(\sigma_s G)\,\mathcal{L}(G\vec{\pi}^2) = $$

$$= \frac{-2(2m_s^2 Z - m_{\eta_s}^2) f_s}{Z^{1/2} f_g} \; \frac{1}{m_G^2 - m_{\sigma_s}^2} \; \frac{2m_u^2 Z - m_\pi^2}{f_g} \sigma_s \vec{\pi}^2 \equiv a \sigma_s \vec{\pi}^2 , \qquad (9)$$

and the corresponding decay width is

$$\Gamma_{\sigma_s \to 2\pi} = 3\frac{a^2}{8\pi m_{\sigma_s}}\sqrt{1 - \left(\frac{2m_\pi}{m_{\sigma_s}}\right)^2} = 26\ \mathrm{MeV} \quad \text{if} \quad \sigma_s = S^*(975) .$$

Hence, the third equation for the fixation of the last parameter f_g appears

$$f_g^2 m_{\sigma_s}^2 = f_g^2 m_G^2 + 2\frac{f_s}{Z^{1/2} a}(2m_s^2 Z - m_{\eta_s}^2)(2m_u^2 Z - m_\pi^2) = \tag{10}$$

$$= 0.06 \text{ GeV}^4 - 0.007 \text{ GeV}^4 = 0.053 \text{ GeV}^4.$$

As a result, we obtain (from(6) and (10))

$$f_g = 212 \text{ MeV} = 2.28 f_\pi,$$

$$m_G = 1.16 \text{ GeV}. \tag{11}$$

The mixing (7) gives us the following values for the masses $m_{\sigma_u'}$, $m_{\sigma_s'}$ and $m_{G''}$:

a) The mixing $(\sigma_u G)$ takes place with the angle $\theta_1 = -22°$ and yields the mass values $m_{\sigma_u'} = 1.07$ GeV and $m_{G'} = 1.17$ GeV.

b) The mixing $(\sigma_u G)$ takes place with the angle $\theta_2 = -8°$ and yields the mass values $m_{\sigma'} = 550$ MeV and $m_{G''} = 1.18$ GeV. It is possible to compare the numbers we have obtained with the well-known experimental data for the scalar meson ϵ (700), $S^*(f_0(975))$, $\epsilon'(f_0(1200))$ masses

$$m_{\sigma_u'} \approx m_\epsilon = 500-700 \text{ MeV} \text{ /4/},$$

$$m_{\sigma_s'} \approx m_{S^*} = 975 \text{ MeV} \text{ /3/}, \tag{12}$$

$$m_{G''} \approx m_{\epsilon'} = 1200 \text{ MeV} \text{ /8/}.$$

Let us verify now with what probability the states in question decay into 2π and 2K. The $\sigma_u \to 2\pi$ decay width has been calculated in /1/. The amplitude of this decay, following from (1), equals

$$T^{(1)}_{\sigma_u \to 2\pi} = 2m_u g Z \sigma_u \vec{\pi}^2 . \tag{13}$$

The inclusion of the intermediate G state changes just a little the original value of $T^{(1)}$

$$T^{(2)}_{\sigma_u \to G \to 2\pi} = -\frac{2f_\pi}{Z^{1/2} f_g^2}\frac{(2m_u^2 Z - m_\pi^2)^2}{(m_G^2 - m_{\sigma_u}^2)}\sigma_u \vec{\pi}^2 = -0.06\, T^{(1)}_{\sigma_u \to 2\pi} . \tag{14}$$

As a result we get for the $\sigma_u \to 2\pi$ decay width

$$\Gamma_{\sigma_u \to 2\pi} = 3Z^2 \frac{m_u^2}{m_{\sigma_u}} \sqrt{1-\left(\frac{2m_\pi}{m_{\sigma_u}}\right)^2}\,(0.94)^2 = \begin{cases} 640 \text{ MeV } (m_{\sigma_u} = 550 \text{ MeV}) \\ 530 \text{ MeV } (m_\epsilon = 700 \text{ MeV}). \end{cases} \quad (15)$$

There are two values of the width for two possible values of ϵ meson mass — the model value $m_{\sigma_u} = 550$ MeV and an experimentally possible one $m_\epsilon \approx 700$ MeV. We can see from (15) that the σ_u decay width is close to the mass of the last mentioned state, which corresponds to the experimental situation /4/.

According to the decay $S^* \to 2\pi$, the value of f_g is fixed, and that is why here the complete agreement with the experiment should be by definition.

Finally, the $G(\epsilon') \to 2\pi$ decay goes in the direct way (see formula (8)) as well as via the intermediate σ_u state

$$T^{(2)}_{G\to\sigma_u\to 2\pi} = -\frac{2f_\pi}{Z^{1/2} f_g}\,\frac{2m_u^2 Z - m_\pi^2}{m_{\sigma_u}^2 - m_G^2}\, 2m_u g Z G \vec{\pi}^2 . \quad (16)$$

For the total amplitude we have as a result

$$T^{(1)+(2)}_{G\to 2\pi} = \frac{2m_u^2 Z - m_\pi^2}{f_g}\left[1 + \frac{(2m_u)^2}{m_G^2 - m_{\sigma_u}^2}\right] G\vec{\pi}^2 = 1.3\, \vec{T}^{(1)}_{G\to 2\pi} , \quad (17)$$

which leads to the width $\Gamma_{G(\epsilon(1200))\to 2\pi} = 150$ MeV in agreement with the experimental data /3,8/. The amplitude of the $G\to 2K$ decay, following from the Lagrangian (8), leads to the width

$$\Gamma_{G(\epsilon')\to 2K} = 30 \text{ MeV}, \quad (18)$$

which is in qualitative agreement with the experiment /3/. The process being taken into account via the intermediate $\sigma_s(S^*)$ state increases twice the value in question.

The calculations we have performed show that after the introduction of the gluonium into the quark model of superconductivity type there are three isoscalar states arising in the scalar sector: ϵ (550), S^*(1070) and ϵ'(1200). In the ground state the lowest resonance consists of the light u and d quarks and it has the large width of the decay into two pions equal to its mass. This resonance is a good candidate for the role of the well-known σ-particle. It is difficult to observe this state because of its large decay width.

The second resonance is close to the scalar meson S^*(975). It consists almost completely of the s-quarks; however, owing to the small admixture of the gluonium, this resonance is allowed to decay into two pions with the decay width corresponding to the experimental value.

Finally, the last resonance possesses the properties close to the ϵ (1200) meson. This resonance decays mainly into two pions with the decay width not contradicting the last experimental data [3,8]. It decays with the smaller probability into two kaons. There are only $BR(\epsilon' \to 2\pi)$ and $BR(\epsilon' \to 2K)$ not entirely in agreement with the experiment (the theoretical value is three times as small as the experimental one). This situation could be a consequence of the fact besides the gluonium there are also four-quark states contributing to the resonance ϵ (1200) [9], and we have not considered such states in our calculation.

As a whole, the picture we have obtained is in complete agreement with the experiment.

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